

## **Redes neuronales artificiales: efecto asimetría y curtosis en la evaluación de portafolios de inversión**

*Artificial Neural networks: asymmetry effect and curtosis in the evaluation  
of investment portfolios*

*Redes neurais artificiais: efeito de assimetria e curtose na avaliação de  
carteiras de investimento*

**Lilia Alejandra Flores Castillo**

Universidad Tecnológica de la Mixteca, México

[floresaly22@hotmail.com](mailto:floresaly22@hotmail.com)

**Martín Carlos Ramales Osorio**

Universidad Tecnológica de la Mixteca, México

[mramales2000@yahoo.com.mx](mailto:mramales2000@yahoo.com.mx)

### **Resumen**

El nivel de incertidumbre hace cada vez más difícil diversificar el riesgo, por lo que se requieren estrategias de inversión que apoyen la toma de decisiones. El objetivo de la investigación es evaluar modelos de selección de portafolios considerando la media, la varianza, la asimetría y la curtosis como parámetros de decisión de inversión. Metodológicamente se parte del pronóstico de rendimientos de activos financieros, utilizando redes neuronales artificiales para construir un portafolio óptimo mediante la técnica de optimización multiobjetivo; finalmente se observan los cambios en el rendimiento del portafolio. Se utilizan datos de acciones que cotizan en mercados bursátiles de México, Argentina, Brasil, Chile, Perú y Colombia. Los resultados validan la capacidad predictiva de una red neuronal, en la aplicación del pronóstico de los rendimientos de

activos financieros. Los portafolios que se conformaron que incluyen la media, la varianza, la asimetría y la curtosis muestran un mayor rendimiento, en comparación con el modelo de Markowitz; asimismo, queda explícito que existe una mayor probabilidad de obtener rendimientos positivos. Se concluye que mediante la técnica de optimización multiobjetivo empleada es posible obtener portafolios óptimos, al incluir los momentos altos de la distribución de los rendimientos de los activos que conforman el portafolio de inversión, representando este modelo una estrategia para el inversionista.

**Palabras clave:** estrategias de inversión, momentos estadísticos, optimización multiobjetivo, portafolio de inversión, redes neuronales artificiales.

### **Abstract**

The level of uncertainty makes it increasingly difficult to diversify risk, and in this context investment strategies are required to support decision making. The objective of the research is to evaluate portfolio selection models considering the mean, variance, asymmetry and kurtosis as investment decision parameters. Methodologically based on the prediction of yields of financial assets, using artificial neural networks to build an optimal portfolio through multiobjective optimization technique; finally, changes in portfolio performance are observed. Stock data are traded on stock markets in Mexico, Argentina, Brazil, Chile, Peru and Colombia. The results validate the predictive capacity of a neural network, in the application of the forecast of the returns of financial assets. The portfolios that were formed that include the mean, the variance, the asymmetry and the kurtosis show a greater yield, in comparison with the model of Markowitz; also it is explicit that there is a greater probability of obtaining positive yields. It can be concluded that by means of the multiobjective optimization technique used it is possible to obtain optimal portfolios by including the high moments of the distribution of the yields of the assets that make up the investment portfolio, this model representing a strategy for the investor.

**Key words:** investment strategies, statistical moments, multiobjective optimization, investment portfolio, artificial neural networks.



## Resumo

O nível de incerteza torna cada vez mais difícil diversificar o risco, razão pela qual as estratégias de investimento são necessárias para apoiar a tomada de decisões. O objetivo da pesquisa é avaliar modelos de seleção de portfólio considerando a média, variância, assimetria e curtose como parâmetros de decisão de investimento. Metodologicamente, baseia-se na previsão de retornos sobre ativos financeiros, usando redes neurais artificiais para construir um portfólio ideal através da técnica de otimização multi-objetivo; Finalmente, são observadas mudanças no desempenho do portfólio. Os dados de estoque que são cotados em mercados de ações do México, Argentina, Brasil, Chile, Peru e Colômbia são usados. Os resultados valem a capacidade preditiva de uma rede neural, na aplicação da previsão do retorno sobre ativos financeiros. As carteiras que foram formadas que incluem a média, a variância, a assimetria e a curtose apresentam maior desempenho, em comparação com o modelo de Markowitz; Da mesma forma, é explícito que existe uma maior probabilidade de obtenção de retornos positivos. Conclui-se que, utilizando a técnica de otimização multi-objetivo utilizada, é possível obter carteiras ótimas, incluindo os momentos altos da distribuição dos retornos dos ativos que compõem o portfólio de investimentos, representando este modelo como estratégia para o investidor.

**Palavras-chave:** estratégias de investimento, momentos estatísticos, otimização multiobjetivo, portfólio de investimentos, redes neurais artificiais.

**Fecha Recepción:** Septiembre 2016

**Fecha Aceptación:** Enero 2017

## **Introduction**

In an environment of economic globalization the impulse of the liberation of the financial systems has generated the diminution of the barriers for the circulation of the financial, commercial and productive capitals; to such a degree that the development of new electronic communication systems and information processing methods facilitate a greater amount of financial transactions between the different countries in the world. The economic dynamics described makes investors look for alternatives to place resources in various stock markets. Generally, they invest in financial assets such as stocks, bonds, currencies, derivatives, real estate, commodities, among others. However, it is necessary to consider the risk that exists when financial markets are integrated. For example, in the absence of macroeconomic stability, in some countries financial instability can be generated and, therefore, an environment of uncertainty in global capital markets.

This framework of financial uncertainty makes it essential to efficiently manage resources; Therefore, it is important to develop new financial evaluation models that contribute to the optimization of capital management.

Portfolio management starts from the concept of diversification; The objective is to combine different instruments to reduce the risk given a level of performance. For this criterion to be valid, it is necessary that the returns of the assets follow a normal distribution. However, Fama (1965) and Mandelbrot (1963) conclude that this assumption is not met in most financial series.

In the process of selecting an investment portfolio, the classic referent is the portfolio theory of the Nobel Prize for Economics, Harry Markowitz, which considers the average and variance as risk selection criteria. However, there are different approaches that have been developed to quantify the risk in an investment portfolio, for example: a) the delta valuation method, which tries to estimate the variation of the value of a portfolio with a measure of sensitivity of the risk factor's; b) the value at risk, which measures the possible maximum loss expected during a certain time interval; and c) econometric models, such as

GARCH models, to forecast volatilities. It is the task of the specialists in the subject to generate alternative models of evaluation of financial assets that are adapted to the needs of the current conditions in economic and financial terms.

The objective of this research is to evaluate a model of portfolio selection that serves as a reference for efficient decision making at the time of investing. The question then arises: Will a model of portfolio selection that includes the mean, variance, asymmetry and kurtosis as a decision parameter manage to increase the portfolio's performance? Demonstrate that the implementation of an optimization model that includes the high moments of the distribution of the assets will allow obtaining optimal portfolios with higher yields, representing this model a strategy for the investor, is the hypothesis that guides the empirical evidence in this investigation.

As the performance of financial assets is an indicator of the performance of an investment, it is important to know its future behavior, in order to provide more information to the investor in such a way that there are more elements in making investment decisions. That is why the first phase of this work is based on the forecast of the performance of actions with artificial neural networks and in a second phase, based on this forecast, the optimal portfolio is constructed and the changes are observed by including the four statistical parameters. For this purpose, the daily data of shares quoted in the stock market of Mexico, Argentina, Brazil, Chile, Peru and Colombia are used.

### **Administration and evaluation of investment portfolios**

Currently the world economy is affected by various social and political factors, which directly and indirectly affects the profits of entrepreneurs. In this context, entrepreneurs are in need of alternative means of income that represent liquidity for the company in an unfavorable situation. Willmer (2015) states that although risk is inherent in investment activities in stocks and the expected return depends on the degree of risk, it is essential to include more elaborate decision models that abandon traditional methods based on intuition and experience.



In developing markets, the application of these models becomes a vital issue due to the instability of the economic variables and the fear that it generates among investors that they can not materialize the desired returns. Investments, according to Gitman (2009), are essentially any instrument in which funds are deposited with the expectation that it generates positive income and / or retains or increases its value. The diversity of investments grants the opportunity to create investment portfolios, defined by Martínez and Perozo (2010) as a set of instruments, whose objective is to obtain a good return minimizing the risk. This instrument selection technique is known as the Modern Portfolio Theory and is also defined as "a compendium of all investments, which can include cash, sight, short and long term. This portfolio can have both fixed income and variable income assets" (México Bursátil, 2012).

Medina (2003) he affirms that the portfolio theory considers that in investment decisions only the expected return and the risk are taken into account. The first moment of the return distribution is used as an estimate of the expected return, and the variance (or standard deviation) of the return is used as a measure of the risk. In the financial area, the standard deviation is known as volatility. On the other hand, Gitman (2009) proposes the following steps to make the investments: (1) comply with the investment prerequisites; (2) establish the investment goals; (3) adopt an investment plan; (4) evaluate the investment instruments; (5) select the appropriate investments; (6) create a diversified portfolio and (6) manage the portfolio.

Managing the portfolio has as main objective to measure the real behavior of the investments in relation to the expected performance. This variation between the actual behavior and the expected behavior is called risk. Moyer et al. (2005) states that portfolio risk, in general, depends on each of the assets that make up said portfolio. Therefore, financial management refers to the proper use of money, focuses on coordinated work to adequately improve financial resources. Robles (2012) argues that financial managers must perform analyzes of assets that symbolize productivity and financial indicators where it is shown that investments must be made to generate higher rates of return and reducing financial risk. By taking an adequate management of the investment portfolios,

diversification is obtained, which has as its main objective the risk reduction that, as Wachowicz and James (2002) states, is achieved as long as the values are not perfectly correlated and positive.

Another benefit of managing portfolios is that the stages of the administrative process (planning, organization, direction and control) serve as a protective barrier against different scenarios previously analyzed. On the other hand, Pinedo (2015) states that portfolio management is based on the appropriate selection of strategies to apply to predictions of the volatility of financial and economic variables. Therefore, it is very important to carry out the administrative process for the creation and follow-up of investment portfolios, since carrying the process in an adequate manner will be able to obtain greater economic benefits and a reduction of losses. This anticipating the facts that may arise in the stock market.

#### **Asymmetry and kurtosis in the selection of investment portfolio**

As stated above, it is crucial to implement investment strategies that reduce risk and maximize profits; therefore, this section mentions the importance of including asymmetry and kurtosis in the selection of assets as an investment strategy and an alternative way of managing a portfolio. The issue of the incorporation of asymmetry in the selection of an investment portfolio has been analyzed by Samuelson (1970), Konno and Suzuki (1995), Chunhachinda, Dandapani, Hamid, and Prakash (1997), Lai (1991), Leung , Daouk and Chen (2001), Li, Qin and Kar (2010), Kemalbay, özkut and Franco (2011) and Solgi, Bayat and Abbasi (2017), who consider that the series of yields do not follow a normal distribution and, given this particularity, it is necessary for investors to incorporate asymmetry as a decision parameter when selecting an investment asset.

In addition, it is important to select the one that shows positive asymmetry because, by increasing the asymmetry, the probability of positive returns increases and the probability of negative returns decreases; in the case of negative asymmetry, the opposite happens. In consideration of kurtosis, Mandelbrot (1963) and Mandelbrot and Taylor (1967) emphasized that it is important to study the excess of kurtosis in financial data, because the phenomenon of heavy tails could be generated. Similarly, when taking into account the



excess of kurtosis, Lai, Yu and Wang (2006), Maringer and Parpas (2009) and Kemalbay, Özkut and Franko (2011) suggested that a positive and high value of kurtosis indicates that the distribution of the yields is leptocurtic and exhibits heavy tails; therefore, a higher frequency of results is presented at the negative and positive ends of the distribution curve. This implies that there is a greater probability of obtaining observations that are very far from the average and can be presented in the best of cases with a greater probability of large profits or, otherwise, large losses in an investment and, therefore, greater volatility; therefore, it is preferable to minimize the value of kurtosis. It should be assumed then that it is important to incorporate the four moments of the distribution of the returns of an investment asset in the selection of a portfolio.

### **Artificial neural networks**

The study on the subject of artificial neural networks becomes important since it represents a useful tool that allows the solution of non-linear problems; they are classified within non-parametric models; The learning is based on determining the relationship that exists between the input data and the output data. The theoretical basis that supports the application of neural networks conforms to the contributions of McCulloch and Pitts (1943), who managed to establish a relationship between the structure and function of neuronal activity; this scoop allowed them to design an artificial network with a simple structure characterized by binary devices with fixed thresholds; Rosenblatt (1961) and Stark, Okajima and Whipple (1962) with competitive learning; Widrow (1962) presented a network formed only by a neuron, the rule of madaline that has multiple elements of adaptation; Werbos (1974) designed the backward propagation algorithm; Grossberg (1976) developed the theory of adaptive resonance that represents a model to identify patterns; Hopfield (1982) presented Hopfield's algorithm, which is a continuation of Hebb's work (1949); Rumelhart, Hinton and Williams (1985) incorporated non-linear elements in the sigmoid activation function of the neurons after the training algorithm was generalized to incorporate multiple layers.

Neural networks have been applied to the classification, identification of patterns, coding and simulation of information. The most well-known application in finances of an artificial neural network is the identification of behavior patterns of economic and financial variables with the purpose of making a forecast of them. The main structure that has a neural network that is used for forecasting is the multilayer perceptron; said architecture consists of an input layer, a hidden layer and an output layer. The input layer allows entering the information that feeds the network, and the number of neurons in this layer is defined by the number of independent variables that explain the behavior of the dependent variable. In the hidden layer is where the training process of the network starts, it has a rule of propagation between neurons and an activation function for each neuron. The output layer is assumed to be linear and is calculated as the linear combination of the response of each of the neurons in the hidden layer, which represents the prognosis of the dependent variable.

## **2. Method**

In this study, a type of descriptive research with a quantitative approach was adopted. For the creation of the hypothetical portfolios, information was used on the closing prices of shares listed on various stock exchanges in Latin America, on a daily basis. The selected actions are CAP (Chile), CASAGRC1 (Peru), ISA (Colombia), BMA (Argentina), CPLE3 (Brazil), GrumaB (Mexico), AsurB (Mexico), GAPB (Mexico), Pe & oles (Mexico), Pinfra (Mexico); the information was obtained from the Economatica database. The time horizon considered includes information from January 3, 2012 to February 15, 2017. The series were standardized by eliminating the days when there is no stock market operation; the yields were obtained by applying the natural logarithm of the stock price  $R_t = \ln (P_t / P_{t-1})$ . As an analytical instrument, the fundamental principles of the investment portfolio theory implemented by Harry Markowitz (1952) were assumed as a reference, and descriptive statistics and the mathematical method of multiobjective optimization were also used.

Regarding the procedure followed in the investigation, this consisted in a first phase in including the future behavior of the returns on financial assets. The forecast of the yields of the actions considered in the investment portfolio was made using an artificial non-linear autoregressive neural network with exogenous input, for which the methodology defined by Kaastra and Boyd (1996) was followed: 1) selection of the variable to predict; 2) sample collection; 3) data processing; 4) network routing, testing within the sample and validation outside the sample set; 5) identification of hidden layers, number of neurons, number of network outputs and transfer function; 6) establish the evaluation criteria; 7) identify the number of training iterations when defining the learning rate and the 8) implementation of the network.

To forecast the returns on financial assets, the topology of the network used in the forecast was a non-linear autoregressive multilayer perceptron with exogenous inputs (NARX), recommended in the works by Chaudhuri and Ghosh (2016), in which a forecast is made of the Indian rupee exchange rate / dollar; Hayes and Prodanovic (2016), who obtained the forecast of energy demand in order to improve the planning and supply of the same; and from Hatata and Eladawy (2017), which by predicting the presence of harmonics in electric power that affect the quality of energy in low voltage electrical systems in Western Egypt, identified that this type of network is highly efficient in the prognosis and has the ability to be trained to perform the forecast of a series of time given the past values of the same.

The network was trained under the directive of supervised learning. In the training process of the network, a vector of input patterns (consisting of random numbers) and a vector with the objective values that in this case were the historical returns of each of the assets were presented. The training of the network was generated by the Levenberg-Marquardt backward propagation algorithm, an optimization algorithm that made it possible to reach a global minimum more quickly and obtain the weights that minimize the adjustment error.

The input vector was compared with the target vector and the error was obtained by least squares. The error propagated backwards throughout the network; the value of the weights was adjusted according to the generated error. Different values were considered for the number of neurons in the hidden layer and in the number of lags included in the structure of



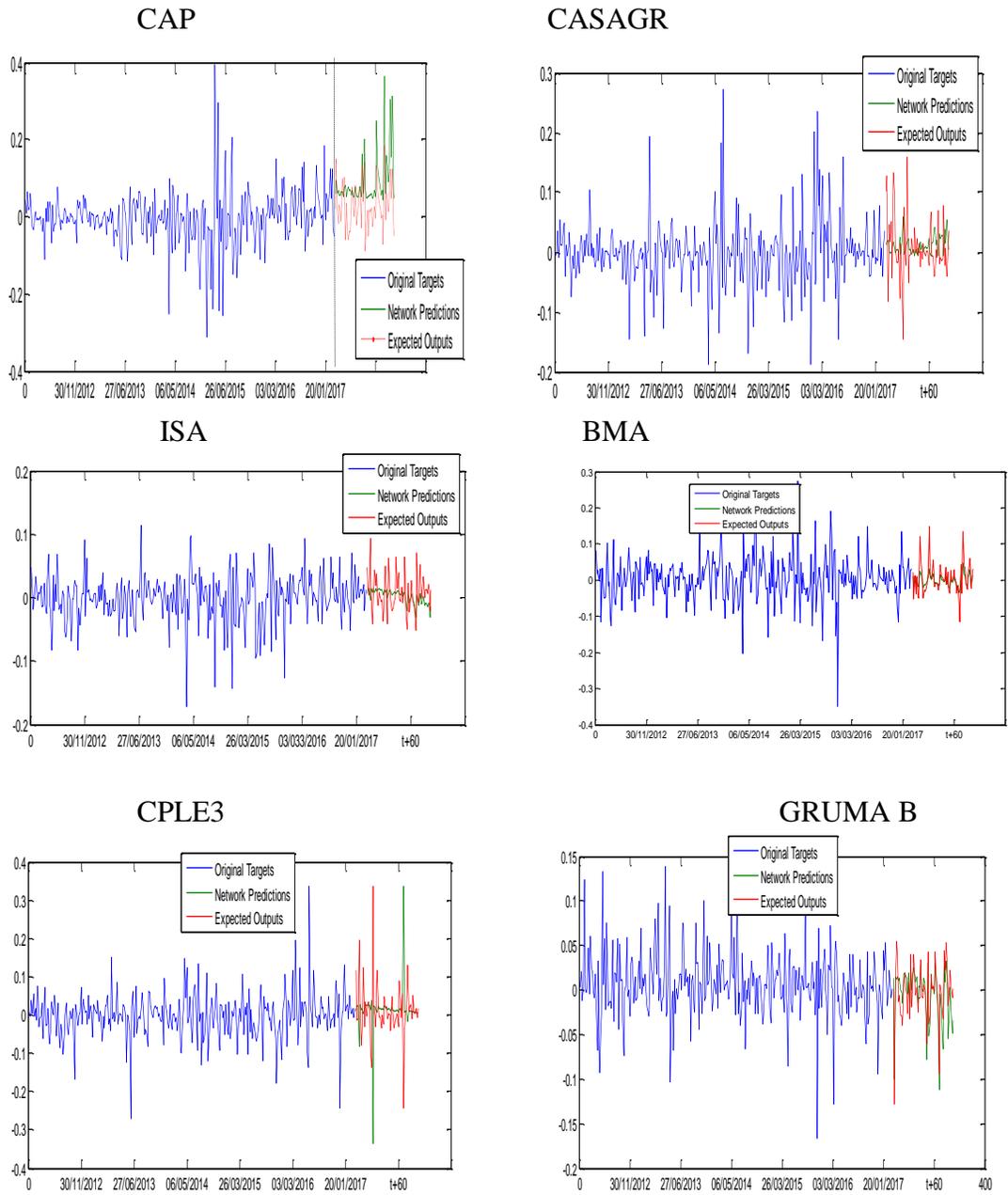
the network and from an iterative trial-error process; this procedure was repeated again and again until the error was minimized and the best performance of the network was achieved. The input data were randomly divided into three subsamples: 70% is used in the training phase, 15% in the validation and measurement of the generalization of the behavior of the neural network and 15% for evaluation outside the sample. Based on the training phase of the network, it was determined that the network structure that minimizes the error is that which considers 60 lags and eight neurons in the hidden layer. Which indicates a better predictive capacity, in relation to the other designed network structures. Once the structure of the network was determined, 60 forward values were predicted.

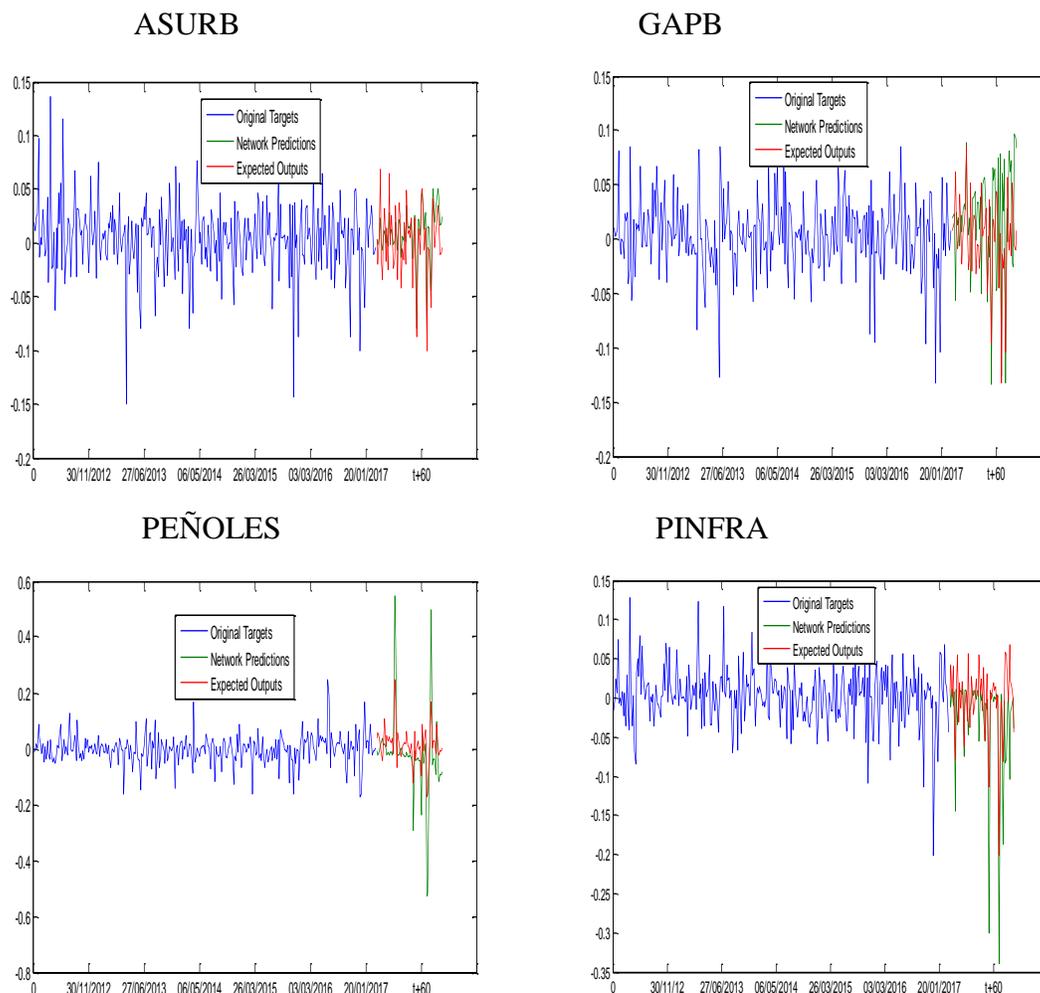
In a second phase, the estimated yields were used to obtain the optimal portfolio, which was obtained by posing a multiobjective optimization problem, where the objective function corresponded to a polynomial that incorporated the individual objectives that were satisfied simultaneously, and additional investor preferences were incorporated.

### **3. Results**

The first result is the forecast of the yields of the shares. The estimated forecast of the performance of each of the financial assets is shown below in Figure 1.

**Figure 1.** Forecast of the returns on financial assets.





Source: elaboración propia con datos de Economática.

In Figure 1 it is observed that the prognosis within the sample for each one of the assets is well adjusted to the objective values (represented by the blue line); however, once the forecast is evaluated outside the sample, the forecast  $t + 60$  (represented by the green line) follows the trend of the real data but has a certain margin of error. This shows the predictive capacity of a neural network and coincides with the results reported by Villada, Muñoz and García-Quintero (2016) who assessed this predictive capacity in the case of the price of gold in periods of high financial stress, considering the crisis US mortgage company in 2008 and determined that a neural network is able to predict with a minimum of error the price of gold.

In the case of Serpa, Muguértégui and Beteta et al. (2016), when comparing the sales forecast using the time series methods, the linear regression and the ANN model, confirmed that a neural network has greater accuracy in the forecast compared to the time series and linear regression models.

Next, an investment strategy is defined based on the forecast obtained through the neural network. The construction of the efficient portfolio was carried out giving solution to a problem of multiobjective optimization, by means of which the proportion of wealth that must be invested in each asset of the portfolio is determined. Multiobjective optimization is the technique that seeks to satisfy simultaneously a set of objectives so that together the best solution is found. It is based on the fundamental principles of the portfolio theory developed by Harry Markowitz basing investment decisions on the risk and return on financial assets and extending to consider both asymmetry and kurtosis as decision parameters. Table 1 shows the value of the statistical moments for each of the financial assets.

**Table 1.** Parámetros estadísticos de los activos financieros

	<i>Rendimiento</i>	<i>Desvest</i>	<i>Asimetría</i>	<i>Kurtosis</i>
CAP	0.32%	0.03	0.97	3.72
CASAGR	169.11%	0.55	0.57	0.94
ISA	62.62%	0.27	-0.05	1.47
BMA	35.29%	0.41	0.29	2.27
CPLE3	89.86%	0.57	-0.10	3.80
GRUMAB	-2.56%	0.33	-0.06	0.97
ASURB	19.26%	0.34	0.32	3.46
GAPB	8.13%	0.35	-0.07	3.46
PEÑOLES	95.64%	0.51	-0.38	2.48
PINFRA	-21.66%	0.39	-0.70	4.45

Source: elaboración propia con datos de Economatica.

Based on the forecast obtained for each of the assets, the mean, variance, asymmetry and kurtosis were determined, parameters considered in this work for investors' decision making in the investment portfolio.

Referring to Table 1, the statistical analysis describes that the assets with negative performance are GrumaB (Mexico) and Pinfra (Mexico), while those with the highest variance are CASAGRC1 (Peru), Pe & oles (Mexico) and CPLE3 (Brazil). Those that are more likely to obtain negative yields according to the parameter of asymmetry are ISA (Colombia), CPLE3 (Brazil), GrumaB (Mexico), GAPB (Mexico), Pe & oles (Mexico) and Pinfra (Mexico). Concerning kurtosis, it is shown that each of the assets has a value of kurtosis different from the one that characterizes a normal distribution, being Pinfra (Mexico) the one that presents an excess of kurtosis. The multi-objective optimization methodology starts from obtaining the statistical moments of the portfolio. The way in which each of them is obtained is expressed in the following equations:

$$\text{Rendimiento del portafolio, } R(w) = W^T R \quad (1)$$

$$\text{Varianza del portafolio, } V(w) = W^T \Sigma W \quad (2)$$

$$\text{Asimetría del portafolio, } S(w) = E[W^T(\tilde{R}-\bar{R})]^3 \quad (3)$$

$$\text{Curtosis del portafolio, } K(w) = E[W^T(\tilde{R}-\bar{R})]^4 \quad (4)$$

Where  $W^T$  indicates the transposed vector of the weights of the portfolio positions,  $R$  the yield of the asset and  $\Sigma$  define the variance matrix covariance for the returns of  $n$  assets.

The next step is to optimize each parameter individually.

$$\text{Maximizar } R = W^T R$$

$$\text{Minimizar } V = W^T \Sigma W$$

$$\text{Maximizar } S = E[W^T(\tilde{R}-\bar{R})]^3$$

$$\text{Minimizar } K = E[W^T(\tilde{R}-\bar{R})]^4 \quad \text{Sueto a } W^T I = 1, W \geq 0 \quad (5)$$

For the hypothetical portfolio it is necessary to  $R^* = 3.05$ ,  $V^* = 0.001$ ,  $S^* = 0.59$  y  $K^* = 2.03$ ;  $M^*$ ,  $V^*$ ,  $S^*$  y  $K^*$  they represent the best scenario or the desired levels, for the mean, variance, asymmetry and kurtosis. The structure of the function that is minimized by the multi-objective optimization considers the deviation of the parameters of the values of the individual functions  $R(w)$ ,  $V(w)$ ,  $S(w)$ ,  $K(w)$  respect by  $R^*$ ,  $V^*$ ,  $S^*$  y  $K^*$ . The function is enriched by adding the preferences of the investors in relation to each objective; the multipliers are used  $\lambda$  to establish the preferences of the investor on the mean, variance and asymmetry of the returns (Leung, Daouk y Chen, 2001; Kemalbay, Özkut y Franco, 2011):

$$\begin{aligned} \text{Min } Z &= \left| \frac{d_1}{M^*} \right|^{\lambda_1} + \left| \frac{d_2}{V^*} \right|^{\lambda_2} + \left| \frac{d_3}{S^*} \right|^{\lambda_3} + \left| \frac{d_4}{K^*} \right|^{\lambda_4} \\ \text{s.t. } W^T R + d_1 &= M^* \\ W^T \Sigma W + d_2 &= V^* \\ E[W^T (\tilde{R} - \bar{R})]^3 + d_3 &= S^* \\ E[W^T (\tilde{R} - \bar{R})]^4 + d_4 &= K^* \\ W^T I = 1, W \geq 0, d_i \geq 0, i &= 1, 2, \dots, n \end{aligned} \quad (6)$$

The solution of the optimization problem is achieved by minimizing the multi-objective function, in such a way that the amount to be acquired of each asset is determined; consequently, the optimal portfolio is obtained.

**Table 2.** Composición de los pesos del portafolio.

	1,1,0,0	1,1,1,0	1,1,1,1
CAP	9.15%	15.30%	0.00%
CASAGR	8.49%	10.62%	18.92%
ISA	45.72%	38.99%	43.79%
BMA	10.42%	12.40%	2.06%
CPLE3	0.00%	0.00%	0.00%
GRUMAB	20.04%	17.50%	35.23%
ASURB	0.00%	4.99%	0.00%
GAPB	6.19%	0.21%	0.00%
PEÑOLES	0.00%	0.00%	0.00%
PINFRA	0.00%	0.00%	0.00%
Rendimiento	53.98%	59.54%	59.25%
Varianza del portafolio	0.0002	0.0002	0.0002
Asimetría	0.13	0.23	0.07
Curtosis	1.74	1.87	1.21

Source: elaboración propia con datos de Economática.

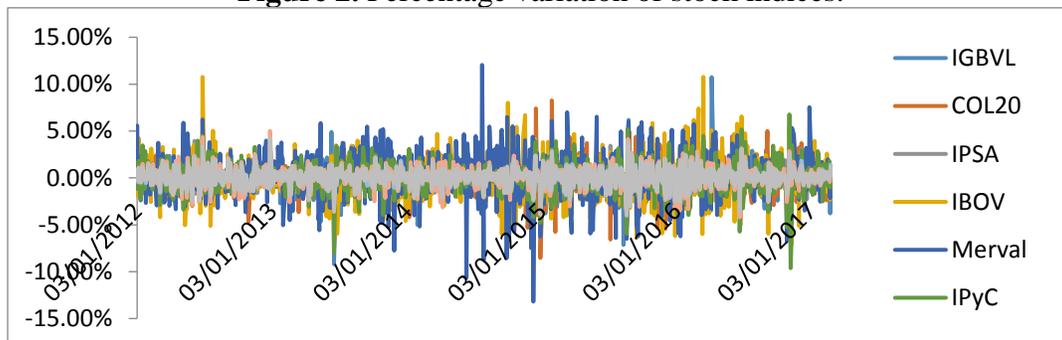
Table 2 shows the percentage that must be invested in each of the investment portfolios. When the investor's preferences are (1,1,0,0) it implies that the investor wants to maximize the returns and minimize the variance; this portfolio represents the Markowitz methodology.



The preferences (1,1,1,0) indicate that the objective is to maximize returns, asymmetry and minimize the level of risk. If the priority is to include kurtosis in the portfolio selection, the preferences would be  $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$  y  $\lambda_4 = 1$  es decir (1,1,1,1).

Of the portfolios shown in Table 2, those who consider the incorporation of asymmetry and kurtosis have higher performance. In general, each of the portfolios present high yields, with a minimum variance and positive asymmetry, which would indicate a greater probability of obtaining positive returns. In order to have a benchmark and measure the efficiency of portfolio management, the stock index of each country from which an asset was selected to integrate the portfolio is considered as Benchmark; the General Index of the Lima Stock Exchange (IGBVL) representative of Peru, Col20 (Colombia), IPSA (Chile), IBOV (Brazil), Merval (Argentina), IPyC (Mexico), Nasdaq (EU) and Standard & Poors 500 (EU). Considering that each of the indices will move in accordance with the average variation of the price of the shares that make up the observed index, Figure 2 shows that in general the indices have reached from a very low 0.19% to 12.5% of performance, also presenting decreases of up to 13.19%.

**Figure 2.** Percentage variation of stock indices.



**Table 3.** Average annual yield of stock indices

AÑO	IPyC	DJIA	NASDAQ	S&P 500
2014	0.10%	0.03%	0.06%	0.05%
2015	-0.08%	-0.01%	0.03%	0.00%
2016	-0.05%	0.06%	0.03%	0.04%
01/03/2017	0.20%	0.18%	0.24%	0.18%

Fuente: elaboración propia con datos de Economatica.

Table 3 provides information on the average annual variation of each index, which in general is very low or nil in some cases. This situation may be due to the events of the world economy (the fall of the Shanghai Stock Exchange is one of the main causes of low, zero or negative returns that occurred in the years 2015-2016, in addition to speculation on the exit of the United Kingdom from the European Union and the impact of the United States elections in 2016), inflation, interest rates and the exchange rate (on May 3, 2017, the Mexican Stock Exchange closed at the low after the announcement of monetary policy of the Federal Reserve of the United States), among other factors. If we consider the average annual Performance of the Stock Indices, IPyC (0.20%), DJIA (0.18%), NASDAQ (0.24%), Standard & Poor 500 (0.18%) and compare it with the performance offered by hypothetical portfolios, It can be said that any of the three portfolios performs well compared to the Benchmark.

#### 4. Conclusion

The portfolio theory considers it important to include parameters such as performance, variance, even the correlation between the assets in the asset selection analysis, thus obtaining a diversified investment portfolio. However, at present it has been observed that when financial collapse occurs in the Stock Exchange of a certain country, adverse effects have been triggered in the global economy and with this the level of volatility has increased; therefore, it is almost impossible to avoid financial risk.

It is important to design financial, statistical models that serve as a basis for investment decision making. According to the results, the forecast of the returns of each of the assets

that make up the investment portfolio was obtained, consolidating the predictive capacity of a neural network in the field of finance and as an effective tool when the observations do not follow a normal distribution. Based on this forecast, the multi-objective optimization methodology was validated as a tool for selecting portfolios by incorporating various parameters such as mean, variance, asymmetry and kurtosis.

Answering the question that guides this research: Will a model of portfolio selection that includes the mean, variance, asymmetry and kurtosis as a decision parameter manage to increase the portfolio's performance? The results indicate that the incorporation of asymmetry and kurtosis is a way to improve the results obtained by the traditional methodology that only considers the mean and variance as decision parameters. In general, investors are expected to prefer positive asymmetries and adverse behavior to kurtosis; On the other hand, the modeling of the two additional moments can improve the performance of the market risk measures. Similarly, it is possible to consider the preferences of the investor for each of the parameters; For example, in case the investor prefers to minimize the risk, a higher weight can be assigned to the variance parameter within the model, which reflects the flexibility of the model when defining the different investment strategies, considering the preferences of the investor.

Faced with a globalized and increasingly volatile economy, it is necessary to make technological adjustments to the traditional portfolio theory methodology to propose new methods, disseminate scientific knowledge among students, teachers, businessmen, business managers, governmental institutions (national, state and municipal), individuals and morals, in order to provide more information regarding new techniques and tools for management and evaluation of investment portfolios that give greater certainty in decision making, minimizing risk and maximizing performance, obtaining greater wealth, which favors the development and economic growth of individual companies and investors, from the aforementioned it is considered important to apply the topic that was addressed.

One limitation of the model exposed in the topic is that the level of risk can be minimized; However, in times of financial uncertainty it is impossible to eliminate the risk in its entirety.



Finally, an expectation for the future is to continue promoting investment portfolio optimization techniques based on avant-garde techniques of risk management.

## Bibliography

- Arroyo, A., & Cossío, F. (2015). Impacto fiscal de la volatilidad del precio del petróleo en América Latina y el Caribe. *Documentos de Proyecto (LC/W.680)*. Comisión Económica para América Latina y el Caribe (CEPAL). Recuperado de: [http://repositorio.cepal.org/bitstream/handle/11362/39706/S1501020\\_es.pdf;jsessionid=EF62C98D196B6FC1285E2BB26ABB166A?sequence=1](http://repositorio.cepal.org/bitstream/handle/11362/39706/S1501020_es.pdf;jsessionid=EF62C98D196B6FC1285E2BB26ABB166A?sequence=1)
- Chaudhuri, T. D., & Ghosh, I. (2016). Artificial Neural Network and Time Series Modeling Based Approach to Forecasting the Exchange Rate in a Multivariate Framework. *Journal of Insurance and financial Management*, 1(5):92-123.
- Chunhachinda, P., Dandapani, K., Hamid, S., y Prakash, A. (1997). Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking & Finance*, 21(2): 143-167.
- Fama, E. (1965). The Behavior of Stock Market Prices. *Journal of Business*, 38(1): 34-105.
- Gitman, (2009). *Fundamentos de inversiones*. México: Pearson Educación.
- Grossberg, S. (1976). Adaptive pattern classification and universal recoding: I. Parallel development and coding of neural feature detectors. *Biological Cybernetics*, 23(3):121-134.
- Hayes, B. P., y Prodanovic, M. (2016). State forecasting and operational planning for distribution network energy management systems. *IEEE Transactions on Smart Grid*, 7(2): 1002-1011.
- Hatata, A. Y., y Eladawy, M. (2017). Prediction of the true harmonic current contribution of nonlinear loads using NARX neural network. *Alexandria Engineering Journal*. Recuperado de: <https://doi.org/10.1016/j.aej.2017.03.050>
- Hebb, D. (1949). *The organization of behavior*. New York: Wiley.
- Hopfield, J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the national academy of sciences*, 79(8): 2554-2558.
- Kaastra, I., y Boyd, M. (1996). Designing a neural network for forecasting financial and economic time series. *Neurocomputing*, 10(3): 215-236.

- Kemalbay, G., Özkut, C., y Franko, C. (2011). Portfolio selection with higher moments: A polynomial goal programming approach to ISE-30 index. *Ekonometri ve Istatistik Dergisi*, (13): 41.
- Konno, H., y Suzuki, K. (1995). A mean-variance-skewness portfolio optimization model. *Journal of the Operations Research Society of Japan*, 38(2): 173-187.
- Lai, T. (1991). Portfolio selection with skewness: a multiple-objective approach. *Review of Quantitative Finance and Accounting*, 1(3):293-305.
- Lai, K., Yu, L. y Wan, S. (2006). Mean-variance-skewness-kurtosis-based portfolio optimization. *First International Multi-Symposium on computer and computational Sciencies (IMSCCS'06)*, 2:292-297.
- Leung, M., H. Daouk, y Chen, A. (2001). Using investment portfolio return to combine forecasts: a multiobjective approach. *European Journal of Operational Research*, 134(1): 84-102.
- Li, X., Qin, Z., y Kar, S. (2010). Mean-variance-skewness model for portfolio selection with fuzzy returns. *European Journal of Operational Research*, 202(1): 239-247.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4): 394-419.
- Mandelbrot, B. y Taylor, H.M. (1967). On the Distribution of Stock Price Differences. *Operations Research*, 15(6): 1057-1062.
- Maringuer, D. y Parpas, P. (2009). Global optimization of higher order moments in portfolio selection. *Journal of Global optimization*, 43(2):219-230.
- Martínez C. y Perozo S. (2010). Sistema de información gerencial para la optimización de portafolios de inversión. *Revista Venezolana de Gerencia*, 15 (50):253-272.
- Medina L. (2003). Aplicación de la teoría del portafolio en el mercado accionario colombiano. *Cuadernos de economía*, 22 (39):128-168.
- México Bursátil, (2012). *¿Qué es un portafolio de inversión? Asesores financieros*. Recuperado de: <http://www.mexicobursatil.com/que-es-un-portafolio-de-inversion/>
- Moyer, R. McGuigan, R. Kretlow, J. y Nuñez, J. (2005). *Administración financiera contemporánea*. México: Thomson.

- Pinedo, O. (2015). Gestión de portafolio de inversión en activos de renta fija. *Revista Moneda*, 164: 24-27.
- Robles, C. L. (2012). *Fundamentos de administración financiera*. México: Tercer Milenio.
- Rosenblatt, F. (1958). The perceptron: a probabilistic model for information storage and organization in the brain. *Psychological review*, 65(6): 386-408.
- Rumelhart, D., Hinton, G., y Williams, R. (1986). Learning internal representations by error propagation. *Letters to nature*, 323(9):533-536.
- Samuelson, P. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *The Review of Economic Studies*, 37(4):537-542.
- Serpa, C., Muguértégui, D., Beteta, J, et al. (2016). Evaluar la capacidad predictiva de los métodos series de tiempo, regression lineal y RNA, poda exhaustiva considerando el mismo margen de error, una aplicación a la demanda. *Revista de investigación Business Intellegence*, 1(1):18-25.
- Sharpe, W. (1966). Mutual fund performance. *Journal of business*, 39: 119-138.
- Stark, L., Okajima, M., y Whipple, G. H. (1962). Computer pattern recognition techniques: electrocardiographic diagnosis. *Communications of the ACM*, 5(10): 527-531.
- Solgi, S., Bayat, M., y Abbasi, Y. M. (2017). The Efficiency of Optimal Portfolio Selection Using Skewness Model. *International Research Journal Applied and Basic Sciences*, 11(1):14-25.
- Villada, F., Muñoz, N. y Garcia-Quintero, E. (2016). Redes Neuronales Artificiales aplicadas a la predicción del precio oro. *Información tecnológica*, 27(5):143-150.
- Wachowicz, J. y James C. (2002) *Fundamentos de administración financiera*. México: Pearson Educación.
- Werbos P. (1974). *Beyond regression: new tools for prediction and analysis in the behavioral sciences* (Tesis de doctorado), Harvard University.
- Widrow, B. (1962). Generalization and information storage in network of adaline 'neurons'. *Self-organizing systems*, 435-462.

Willmer E. (2015) Metodología para la toma de decisiones de inversión en portafolio de acciones utilizando la técnica multicriterio AHP. *Contaduría y Administración*, 60 (2):346-366.

<b>Rol de Contribución</b>	<b>Autor(es)</b>
<b>Conceptualización</b>	Lilia Alejandra Flores Castillo
<b>Metodología</b>	Lilia Alejandra Flores Castillo y Martín Carlos Ramales Osorio
<b>Software</b>	NO APLICA
<b>Validación</b>	Lilia Alejandra Flores Castillo y Martín Carlos Ramales Osorio
<b>Análisis Formal</b>	Lilia Alejandra Flores Castillo y Martín Carlos Ramales Osorio
<b>Investigación</b>	Lilia Alejandra Flores Castillo
<b>Recursos</b>	Lilia Alejandra Flores Castillo
<b>Curación de datos</b>	Lilia Alejandra Flores Castillo
<b>Escritura - Preparación del borrador original</b>	NO APLICA
<b>Escritura - Revisión y edición</b>	Martín Carlos Ramales Osorio
<b>Visualización</b>	Lilia Alejandra Flores Castillo y Martín Carlos Ramales Osorio
<b>Supervisión</b>	Martín Carlos Ramales Osorio
<b>Administración de Proyectos</b>	Martín Carlos Ramales Osorio
<b>Adquisición de fondos</b>	Lilia Alejandra Flores Castillo